लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2019

CMSACOR04T-COMPUTER SCIENCE (CC4)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Construct a truth table for the compound proposition:
$(p\leftrightarrow q)\oplus(\neg p\leftrightarrow q)$.

(b) What is called graph isomorphism?

(c) How many permutations are there in the word "MALAYALAM"?

(d) Define a Semigroup.

(e) Define Big-O Notation.

(f) Give an example of a lattice that is not complemented.

(g) Show that in a Boolean algebra
$ab'+a'b=0$ iff $a=b$.

(h) Find the recurrence relation of the sequence $S(n)=a^n : n\geq 1$.

GROUP-B

Answer any *four* from the following $10\times4=40$

2. (a) Prove that $\sum_{k=1}^{n}\theta(f_k(n))=\theta\left(\sum_{k=1}^{n}f_k(n)\right)$ by using the linearity property of summations. 3+3+4

(b) Show that the solution to $T(n)=2T(\lfloor n/2\rfloor+17)+n$ is $0(n\ lg\ n)$ by using substitution method of solving recurrences.

(c) Solve the following recurrence by using recursion tree:
$T(n)=2T(n/4)+\sqrt{n}$.

3. (a) Prove that the number of different permutations of n objects, where there are n_1 indistinguishable objects of type1, n_2 indistinguishable objects of type 2, ……, and n_k indistinguishable objects of type k, is $\dfrac{n!}{n_1!\ \ n_2!\ldots\ldots n_k!}$. 3+3+4

(b) Suppose that every student in a discrete structures class of 25 students is a freshman, a sophomore, or a junior. Show that there are either at least three freshman, at least 19 sophomores or at least five juniors in the class.

(c) How many positive integers between 100 and 999 inclusive –

(i) are not divisible by either 3 or 4?

(ii) have the same three decimal digits?

4. (a) What is the truth value of the quantification $(\exists x)\ Q(x)$ if the statement $Q(x)$ and universe of discourse is given as follows? 2+4+2+2

$Q(x): x^2 < 12$, where U = {positive integers not exceeding 3}.

(b) Let p denote the statement, 'The weather is nice' and q denote the statement, 'we have a picnic'. Translate the following in English and simplify if possible: -

(i) $\neg q \leftrightarrow \neg p$; (ii) $\neg(\neg p \vee q) \vee \neg(p \wedge \neg q)$

(c) Show that the following argument is a fallacy:

$p \rightarrow q,\ \neg p \vdash \neg q$.

(d) Show that $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$ is a tautology.

5. (a) What is a Multigraph? Describe with an example. 2+2+2+4

(b) What is called the eccentricity of a vertex?

(c) Differentiate between the adjacency and incidence matrix representations of a particular graph by taking an example.

(d) Prove that a simple graph with n-vertices and k-components can have at most $(n-k)(n-k+1)/2$ edges.

6. (a) Prove that a graph is a tree if and only if it is minimally connected. 2

(b) What is a Spanning Tree? Give an example. 2

(c) What is a bipartite graph? What are the properties of Kuratowski's two graphs? 2+2

(d) What is a chromatic number? 2

7. (a) Use mathematical Induction to prove that 4+3+3

$1.1! + 2.2! + \ldots\ n.n! = (n+1)! - 1\ \ \forall n \geq 1$

(b) Solve the recurrence relation

$a_n = -4a_{n-1} - 4a_{n-2}$, $a_0 = 0$ and $a_1 = 1$

(c) Sort the following array by using Bubble Sort

(7, 0, 2, 10, 5)

8. (a) Prove that $\sim(p \wedge q) \equiv \sim p \vee \sim q$ 4+3+3

(b) Suppose repetitions are not permitted. How many 3-digit numbers can be formed from six digits 2, 3, 4, 5, 7 and 9?

(c) Explain valid argument and universal quantifier.

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